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## 1. Introduction

This is the final report of WP5. It summarizes the previous reports, hence it does not offer essential new information. See the bibliography for other reports and papers resulting from this workpackage. The objective of workpackage WP5 is to develop novel, general learning models which do not require the (geo)metric assumption, thereby working directly on the original data. Game theory offers an attractive and unexplored perspective that serves well our purpose. The aim was to develop a game-theoretic framework based on a formalization of the competition between the hypotheses of class membership. According to this perspective, the focus shifts from optima of objective functions to equilibria of (non-cooperative) games.

The workpackage was divided into two tasks, each with its own deliverable:

- Novel equilibrium concepts for pattern recognition and related algorithms.
- Generalizations of game-theoretic notions.

The overall goal of the first task was to develop a game-theoretic framework for clustering and related problems, providing theoretical and experimental and algorithmic analysis of the application of the notion of Nash equilibrium and its refinements in the context of pattern recognition.

The second task addresses several generalizations of this framework. The game-theoretic framework naturally generalizes to allow k-way interactions among players, which is equivalent to using high-order similarity relations (hypergraphs) which can capture invariances that are not available in pairwise relations. A further generalization involves the existence of distinct populations of players, each characterized by its own strategies and payoffs. Finally, we explored the use of non-linear payoff functions, which will allow us to deal with context-dependent similarities.

The main scientific achievements made in the different workpackages are described in the relevant deliverables. However the main contributions (with reference to the published results from the project) are as follows.

## 2. Equilibrium concepts for pairwise clustering, and related algorithms

We explored and extended the grouping framework introduced in (Torsello, Rota Bulò, and Pelillo, 2006), which saw the introduction of a new framework for grouping and clustering derived from a game-theoretic formalization of the competition between the hypotheses of group membership. The basic idea behind the proposal is as follows: the hypotheses that each object belongs to the figure compete with one-another, each obtaining support from compatible edges and competitive pressure from all the other. Competition will reduce the population of individuals that assume hypotheses that do not receive strong support from the rest, while it will allow populations assuming hypotheses with strong support to thrive.

Eventually all inconsistent hypotheses will be driven to extinction, while all the surviving hypotheses will reach an equilibrium. At equilibrium all surviving hypotheses receive the same average support, hence exhibiting the internal coherency of a group, while all the extinct hypotheses must have a lower support, hinting to external incoherency. With such formalization the group corresponds to the evolutionary stable strategies of a non-cooperative grouping game, which are found using replicator dynamics, a classic formalization of a natural selection process.

**Enumeration and overlapping groups.** The first extension regarded the possibility of extracting overlapping clusters in a pairwise context, and it is based on two important properties of the game-theoretical approach. First, the approach works as a multi figure/ground discrimination algorithm, extracting only cohesive groups, while leaving spurious entries unclustered. Second, the clusters are extracted as surviving strategies at an equilibrium, thus different equilibria can provide different, possibly overlapping, groups. Transforming the payoff matrix that drives the evolution of the selection process, we can render unstable previously extracted equilibria, while not affecting any other cluster. This guarantees that once found, a cluster will not be extracted again. The net result of this process is an approach to enumerate all possible groups approximately in order of relevance. This work has been published in (Rota Bulò, Torsello, Pelillo, 2009; Torsello, Rota Bulò, and Pelillo 2008).

**Algorithms.** We explored new efficient algorithms to extract Nash equilibria as a tool to achieve efficient classification. The replicator dynamics used in previous work, and in general all the payoff monotonic dynamics have serious drawbacks for the use in automatic classification. First, the simplex and its faces are invariant under imitation dynamics. This observation has two implications: fixed points under imitation dynamics may not be Nash equilibria, and every trajectory never reaches the boundary of the face from which it started in finite time. This problem forces the need of approximating the support by setting a completely arbitrary threshold to decide whether a strategy has non-zero support. Second, each iteration of the dynamics is quadratic in the number of elements to be classified, leading to computation times that are too high for large scale problems, thus requiring the definition of out-of-sample approaches. Building upon the invasion barrier paradigm, we proposed an Infection and Immunization Dynamics (InImDyn), modelling a plausible adaptation process in a large population. This dynamics exhibits a better asymptotic behaviour compared to other popular procedures like Replicator Dynamics, and can establish support separation in finite time, which can never be achieved by any interior-point method or any other evolutionary game dynamics. This last property is particularly interesting as it eliminates the need for an arbitrary threshold to extract the members of a cluster. This work has been published in (Rota Bulò, Bomze 2009; Rota Bulò, Bomze and Pelillo 2010; Rota Bulò, Pelillo and Bomze 2011).

**Equilibrium concepts for clustering.** An initial investigation of classic game-theoretic concepts of equilibria showed that the vast majority of the method proposed in the literature is

a refinement of the Nash equilibrium, where additional constraints are added to offer stronger guarantees. Experiments on perceptual grouping and image segmentation clearly showed that the Nash equilibrium is already overly-restrictive, often leading to over-segmentation of the data. Motivated by this observation we decided to start our investigation on relaxations rather than refinements of the Nash equilibrium. To this end we developed the concept of maximal good which is defined as the set of strategies whose face is completely contained in the basin of attraction of an evolutionary stable strategy for all payoff-monotonic evolution dynamics. A different starting point of investigation comes from *essential* equilibria, i.e. a refinement of Nash Equilibria that do not vary much as the observed similarities vary. Using this notion we introduced the *robustness* of Dominant Set with respect to noise, which is commonly found in applications.

### 3. High order and contextual grouping

The game-theoretic framework can be naturally generalized to allow k-way interactions among players, which is equivalent to using high-order similarity relations, i.e. hypergraphs.

**Hypergraph clustering and Motzkin-Straus theorem for hypergraphs.** To follow this direction of investigation in (Rota Bulò and Pelillo 2009a), we generalized the Motzkin-Straus theorem relating cliques of a graph to the optima of a quadratic problem on the standard simplex, which is strongly related to the evolutionary stable strategies of our game theoretic clustering formulation. The approach was further generalized in (Rota Bulò and Pelillo, 2009b; Rota Bulò and Pelillo, 2011) to clustering with continuous high-order affinities. The basic idea behind our approach is that the hypergraph clustering problem can be considered as a single-population multi-player non-cooperative “clustering game”. Let  $H$  be a  $k$ -graph modeling a hypergraph clustering problem, where hyperedges are weighted with a similarity function defined on  $k$ -tuples objects. We can build a game involving  $k$  players, each of them having the same set of (pure) strategies , which correspond to the set of objects to cluster. Under this setting, a population of agents playing a clustering game is to all intents and purposes a representation of a cluster. The payoff function of the clustering game is defined in a way as to favour the evolution of agents supporting highly coherent objects. Intuitively, this is accomplished by rewarding the  $k$  players in proportion to the similarity that the  $k$  played objects have. Our generalization links cliques of  $k$ -uniform hypergraphs to the minimizers of a polynomial optimization problem on the standard simplex. The problem is then optimized using a dynamical system that can be seen as a high-degree (or contextual) form of the classical replicator dynamics developed by Baum and Eagon in the late '60s.

**Hypergraph matching.** In (Ren, Wilson, Hancock, 2011) an approach for hypergraph matching was proposed based upon the work of (Rota Bulò and Pelillo, 2009b). The proposed approach is based on the factorization of the compatibility tensor, derived from two hypergraphs to match, in terms of a matrix defined in probability domain. Let  $H$  be a  $k$ -graph, its adjacency tensor is indexed by  $k$ -tuples of vertices and a similarity function provides for each tuple corresponding to an edge of the hypergraph a similarity value. Starting from a

function measuring hyperedge similarity they construct a *compatibility tensor*. Under the conditional independence assumption of the matching process, the hyperedge matching probability can be factorized over the associated vertices of the hypergraphs. High order matching problems can be formulated as locating the matching probability that most closely accords with the elements of the compatibility tensor, i.e., seeking the optimal by maximizing the objective function.

**Feature selection.** In (Zhang and Hancock, 2011a; Zhang and Hancock, 2011b; Zhang and Hancock, 2011c) a game-theoretic approach for feature selection is proposed based upon the work of (Rota Bulò and Pelillo, 2009b). The proposal is based on the idea of clustering features based on the mutual information of the feature vectors and selecting the features that best describe each cluster. With this view, the process is threefold: First, a relevance matrix, or a tensor with triple-wise interactions, must be computed from the feature vectors, then clusters are extracted using the dominant sets framework, and finally the features are selected based on multidimensional interaction information between features and target classes.

**Context dependent similarities.** In (Erdem and Torsello, 2009) we investigated the idea to of learning using contextual-dependent similarities. Typically in similarity-based approaches, the similarity between two shapes is a universal measure, often requiring metric properties as well, of how well the primitives forming the shapes and/or their spatial organizations agree. However the human perception of similarity is not only non-metric, but also strongly dependent on the surrounding context. We studied a game-theoretic approach where we learn both the categories present in the data and the specific intra-category similarities that emerged from the context. We iteratively clustered the shapes, recomputing the similarities based on the extracted class information. The resulting contextual similarity is not symmetric, making the ability of the game-theoretic approach to deal with asymmetric affinities of fundamental importance.

#### 4. Multi-Population formulations

**Graph transduction.** In (Erdem and Pelillo 2011a; Erdem and Pelillo 2011b) we developed a multi-population game-theoretic approach for graph transduction. In this approach distinct populations of players, each characterized by its own strategies and payoffs, was proposed. Graph transduction is a popular class of semi-supervised learning techniques which aims to estimate a classification function defined over a graph of labeled and unlabeled data points. The general idea is to propagate the provided label information to unlabeled nodes in a consistent way. In contrast to the traditional view, in which the process of label propagation is defined as a graph Laplacian regularization, within the proposed game-theoretic framework, consistent labelings of the data correspond to equilibria of the game. An attractive feature of this formulation is that it is inherently a multi-class approach and imposes no constraint whatsoever on the structure of the pairwise similarity matrix, being able to

naturally deal with asymmetric and negative similarities alike. The theoretical motivation for the proposed approach stems from the analysis of the simplest case of graph transduction where the graph expressing the similarity relationships among the data points is an unweighted undirected graph. Following this observation, we can formulate graph transduction as a (continuous) constraint satisfaction problem. Experiments on a number of real-world problems demonstrate that the proposed approach performs well compared with state-of-the-art algorithms and it can deal effectively with various types of similarity relation.

**Consensus and probabilistic clustering.** Multi-population approaches has been applied to the problem of consensus clustering, i.e, the problem of finding a proper consensual aggregation of data objects from a number of different input clusterings which have been obtained for a particular dataset. The proposed approach is built upon the evidence accumulation framework: For each pair of data objects, the probability of them to be clustered together (co-occurrence probability) is estimated from the ensemble of clusterings. This yields a co-occurrence matrix; the idea behind the consensus clustering approach is that the co-occurrence matrix can be factorized in terms of hidden probabilistic assignments of data points to  $K$  classes. These assignments reside in a  $K$ -dimensional simplex  $\Delta_K$  and the union of all these assignments then lays in a multi-simplex  $\Delta^n_K$ , where  $n$  is the number of objects to be clustered. This work has been published in (Rota Bulò and Pelillo 2010; Rota Bulò, Lourenco, Fred and Pelillo 2010).

**Correlation clustering.** In (Rebagliati, Rota Bulò and Pelillo, 2011) we applied the multipopulation formulation in the context of Correlation Clustering. We developed a heuristic algorithm, based on the Baum-Eagon inequality, for the Correlation Clustering problem, returning a partition solution in the form of an assignment matrix  $Y$  in  $\Delta^n_K$ . The main advantage of this model is that the number of classes does not need to be fixed in advance. However we proved that Correlation Clustering formulation cannot explicitly capture overlapping clusters. In order to overcome these limitations we adapted the functional of (Rota Bulò, Lourenco, Fred and Pelillo, 2010) in this setting. Additionally, we propose a simple way for building an ensemble of weak hyperplane classifiers sampled from a reproducing kernel Hilbert space, which allows to apply Correlation Clustering without the empirical estimation of pairwise correlation values. Preliminary experiments on datasets from the UCI repository confirm enhanced capabilities of this framework.

**Semantic image labelling.** A similar multi-population approach has been applied to the problem of semantic image labelling, i.e., the problem of assigning object class labels to all pixels in a test image. We provide an interpretation in terms of a label puzzle game by considering semantic image labelling as the task of assembling possibly overlapping label puzzle pieces, where the pieces are label configurations obtained by means of a modified random forest classifier. This work has been published in (Kontschieder, Rota Bulo', Donoser, Pelillo and Bischof, 2011; Kontschieder, Rota Bulo', Bischof and Pelillo, 2011).

## 5. Applications

**Matching and parameter estimation.** The Matching problem can be formulated as a competition between correspondence hypotheses and the selection process leads to an equilibrium where only compatible correspondences survive. This matching process can then be used as inlier selection for robust parameter estimation. One of the most interesting property of this approach is its selectivity, i.e., its tendency to select small sets of highly compatible elements. This property derives directly from the simplex constraint of the game-theoretic estimation approach, and finds a theoretical justification in the fact that the Lagrangian of the simplex is equivalent to an  $L_1$  regularization. However, since the constraint is enforced in a hard way, the game-theoretic competition drives for an even stronger sparsity of the solution than what is obtained with the lasso constraint. This is particularly evident in graph-matching tasks, where in contrast to the traditional quadratic assignment-based formulation which favors the largest possible set of consistent matches, the game-theoretic matcher selects a sparse set of very good correspondences, thus favoring low false positive rather than low false negative matches. This property made the approach particularly attractive for inlier selection problems, where the presence of outliers severely affect the final estimation. In these situations a common approach is to use ex post filtering approaches like RANSAC, which fail with a very strong or structured noise. In contrast, with the use of a game-theoretic approach, the evolution of the population drives the selection towards a highly cohesive subset of inliers.

This framework initiated with (Albarelli, Pelillo, and Viviani, 2008; Albarelli, Rota Bulò, Torsello, and Pelillo, 2009) and has found several applications in pose estimation from stereo images (Albarelli, Rodolà, and Torsello, 2010a; Albarelli, Rodolà, and Torsello, 2011b), semi-local geometric models (Rodolà, Albarelli, and Torsello, 2010a; Rodolà, Albarelli, and Torsello, 2010b), surface registration with isometry estimation in (Albarelli, Rodolà, and Torsello, 2010b; Albarelli, Rodolà, and Torsello, 2011c), and recognition of objects from cluttered 3D scenes (Albarelli, Rodolà, and Torsello, 2011a; Rodolà, Albarelli, and Torsello, 2011c) .

**Feature selection.** The selectivity of the game-theoretic approach and its characteristic of being a one-class clustering framework can be used to perform relevant feature selection. In (Albarelli, Rodolà, Cavallarin, and Torsello, 2010) this approach was applied to the problem of extracting and matching a signature from a cluttered and textured background, whereas in (Albarelli, Rodolà, and Torsello 2010c) the idea was applied to the selection of robust surface descriptors for 3D registration.

**Feature combination.** Feature combination is an effective method for improving object recognition and classification performance. Feature combination methods can be categorized into two types according to the level at which they operate. The first one use features of all individual classifiers to form a joint feature vector, which is then used in later classification. In

the case of Support Vector Machine (SVM) classification, feature combination translates to combining a set of kernel functions into one final kernel function. The second type operates at the decision or the score level, namely, the outputs of all individual classifiers are used in combination. This approach is attractive as different types of classifiers, e.g., SVM and kNN, can be combined together. In this paper we focus on kernel combination with application in SVM classification. We developed a simple yet effective weighting scheme for feature combination based on the dominant set concept. Specifically, we use the dominant set clustering method to evaluate how difficult a kernel matrix is for a SVM classifier. This degree of difficulty is found to be related to the classification performance and thus is used as the weight in feature combination. This work is under preparation in (J. Hou and M. Pelillo, 2011).

## 6. Conclusions

This workpackage developed a novel game-theoretic framework on which to base machine learning and pattern recognition tasks. Game theory offers an attractive and unexplored perspective that offers several advantages over traditional optimization-based approaches, as it is readily applied to any form of second or higher-order relation, without any (geo)metric assumption. The framework has been successfully applied to a variety of problems ranging from unsupervised to semi-supervised learning, to matching, to robust estimation, to feature selection, providing a unifying framework under which to analyse all of these applications and providing, in some cases, a more or less direct generalization of well-known approaches.

These methods are already having a significant impact in the scientific community outside the SIMBAD consortium. In fact, our game-theoretic learning approaches are being used by several groups in such diverse areas as bioinformatics (Frommlet, 2010), video analysis (Sakarya and Telatar, 2010; Asan and Alatan, 2009; Li et al., 2009), brain topography (Dimitriadis et al., 2009), human behavior analysis (Hamid et al, 2009; Hung and Kroese, 2011), object/shape detection and recognition (Yang, Liu and Latecki, 2010; Liu and Yan, 2010a; Liu and Yan, 2010b; Liu, Latecki, and Yan, 2010), image retrieval (Wang et al., 2008), etc. Further, we do believe that the intrinsic properties of our game-theoretic notion of a cluster can have a significant impact in the analysis of social networks or, more generally, in the analysis of complex systems.

Finally, an interesting property of our framework is its selectivity, i.e., its tendency to select small sets of highly compatible elements. This property derives directly from the simplex constraint, and points to an interesting link between competition and L1 regularization. This is an open issue that still remains to be investigated and which might give an insight to the new wave of sparsity enforcing machine learning approaches ranging from lasso regression to compressive sensing.

## 7. Publications resulting from WP5

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- A. Albarelli, E. Rodolà, A. Cavallarin, and A. Torsello (2010), "Robust Figure Extraction on Textured Background: a Game-Theoretic Approach." In 20th International Conference on Pattern Recognition. (ICPR2010).
- A. Albarelli, E. Rodolà, and A. Torsello (2010a), "Robust Game-Theoretic Inlier Selection for Bundle Adjustment." In 3D Data Processing, Visualization and Transmission (3DPVT).
- A. Albarelli, E. Rodolà, and A. Torsello (2010b), "A Game-Theoretic Approach to Fine Surface Registration without Initial Motion Estimation." In IEEE International Conference on Computer Vision and Pattern Recognition (CVPR2010).
- A. Albarelli, E. Rodolà, and A. Torsello (2010c), "Loosely Distinctive Features for Robust Surface Alignment." In 11th European Conference on Computer Vision (ECCV2010), pp. 519-532.
- A. Albarelli, E. Rodolà, and A. Torsello (2011a), "A Non-Cooperative Game for 3D Object Recognition in Cluttered Scenes." In International Conference on 3D Imaging, Modeling, Processing, Visualization and Transmission, pp. 252-259.
- A. Albarelli, E. Rodolà, and A. Torsello (2011b), "Imposing Semi-Local Geometric Constraints for Accurate Correspondence Selection in Structure from Motion: A Game-Theoretic Perspective." *International Journal of Computer Vision*.
- A. Albarelli, E. Rodolà, and A. Torsello (2011c), "Fast and Accurate Surface Alignment Through an Isometry-Enforcing Game." Submitted to IEEE Trans. Pattern Anal. Mach. Intell. (PAMI).
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